How to theorize music today in the light of mathematics?
A musician's point of view

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François Nicolas [3]

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"*Despite all the experience that I could have acquired in Music,*
as I had practiced it for quite a long time,
it's only with the help of Mathematics that I have been able to untangle my ideas,
and that light made me aware of the comparative darkness in which I was before."
Rameau (1722)

If the relationships between mathematics and music can't be limited to their theoretical dimension – I had the opportunity, during the last annual day of SMF [4] "Mathematics and Music" (June 21st, 2008) [5], to suggest that music and mathematics would gain a lot by referring themselves to the *making of* (music from mathematics / mathematics from music) -, it is however obvious, since the meeting Euler-Rameau in 1752 [6], that theorizing music in the light of mathematics is still the most productive approach.

It is clear that two hundred and fifty years after this meeting, the ways of implementing such a type of theorization have significantly changed.
One would like to clarify the new theoretical configuration reached nowadays, at this beginning of XXI° century, by the ten years experience [7] which has been deployed under the denomination "*mamuphi*" (for ***ma****thematics*-***mu****sic*-***phi****losophy*) [8] which includes a seminar (Ens-Ircam-CNRS), a school (of mathematics for musicians), and various meetings or publications [9].
The *mamuphi* nebula gives us a better analysis of the various ways to theorize music today in the light of mathematics and to choose the best mathematical tools for that. In *mamuphi*, these tools converge: they are primarily those of the algebraic geometry as redrawn by Grothendieck [10], and more specifically those of his topos theory [11]. Thus mathematicians, musicologists and musicians converge in *mamuphi* to privilege this toposic approach [12]; however, as one will see it, they diverge from each other about the ways of implementation.

Precise details

- To theorize the music can be done many manners: there are *acoustical, psychological, economical, sociological, ethnological, psychoanalytical*, but also *philosophical, epistemological*, etc, theorizations of the music as there are *mathematical, musicological and musical* one. Only these three last methods are active in *mamuphi*.
- If *mamuphi* registers philosophy in its workspace, it is not primarily for possible philosophical theorizations of the music - such, for example, that which Adorno have produced -; it is rather the following conviction: one can theorize the music in the light of mathematics only in the shade of philosophy (more precisely: in the shade of *a* given philosophy, suitable for the followed orientation). This shade of philosophy is due to the fact that what "theorizing" means does not go from oneself: theorizing does not have a univocal meaning but depends not only on *what* is to be theorized, but very as much of *which* theorizes (say: of its "subject" as much as its "object"). It is at the precise point where these various designs from theoretical should be articulated - these "theoricities" - that philosophy will play its part.

In this article, one will leave side this philosophical aspect of *mamuphi* work.

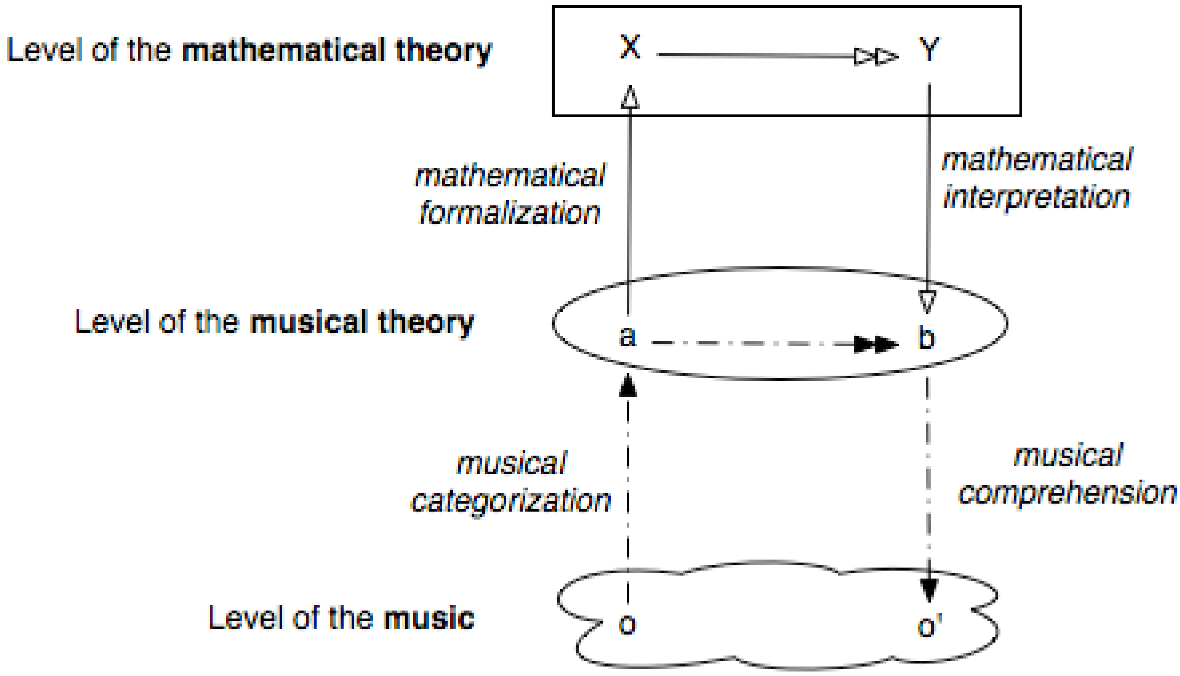
mamuphi confronts three different manners to theorizing the music (in the light of mathematics and in the shade of philosophy): a *mathematician* manner, a *musicological* manner and a *musician* manner.

I. *MATHEMATICIAN* MANNER OF THEORIZING THE MUSIC

This first manner take again, under the contemporary mathematical conditions, the emblem of the great Euler. Today, the work of Guerino Mazzola [13] prolongs this mathematical tradition.
Quite naturally, the musician will find in this Mazzola's work characteristics already present in the Euler's theory of the music. [14]

Theory of a theory

A mathematical theory of the music always leaves a preexistent theory of the music, that this theory (which is used as precondition to the mathematician) be of musical nature (as at the time of Euler) or rather of musicological nature (like today [15] for Mazzola). Indeed a mathematician cannot build his theory directly starting from musical scores (even if he can very well read these scores, the mathematician will hardly plan to propose a new idea of them) but starting from preexistent analyses of these scores, therefore from preformed musical theories which will be used as a basis for its own work.
One can draw the scaffolding of the theories thus [16]:



For example, the Mazzola's theory undertake, in the course of its vast project, to formalize:

- the theory of the counterpoint by Johann Joseph Fux (XVIII° century),
- the theory of the tonal harmony by Hugo Riemann (XIX° century),
- the analysis of the *Hammerklavier* sonata (Beethoven) by two musicologists Ratz & Uhde (XX° century),
- the analysis of *Structures I.a* (Boulez) by Giorgi Ligeti.

To theorize mathematically, it is to formalize, and thus to deform

To mathematically theorize an existing musical theory, it is to formalize it according to own mathematical requirements. This formalization, not being neither a translation nor a simple transposition [17], thus implies a deformation; it requires a rehandling of the original theory so that the categories common to both theoretical faces will have, at the end, very shifted significances.
One can realize it in the eulerian design of consonance/dissonance relationship [18]. One finds this point at Mazzola, for example in his formalization of "cadenzas" and of "modulations": between its mathematical concepts (of *cadenza* and of *modulation*) and the homonymous musical concepts, the relations will be of intersection rather than of recovery. Let us explain that.
For its own needs, Mazzola retains musical modulation only two properties:

- the existence of harmonic sequences able to affirm a particular tonality (those which will articulate a tonal cadenza: for example II-V-I);
- the existence of harmonic sequences which are common to two close tonalities (it acts here to use enharmonic chords, carrying an tonal ambiguity: for example II-I-IV in major C could be reinterpreted like VI-V-I in major F).

But while thus proceeding,

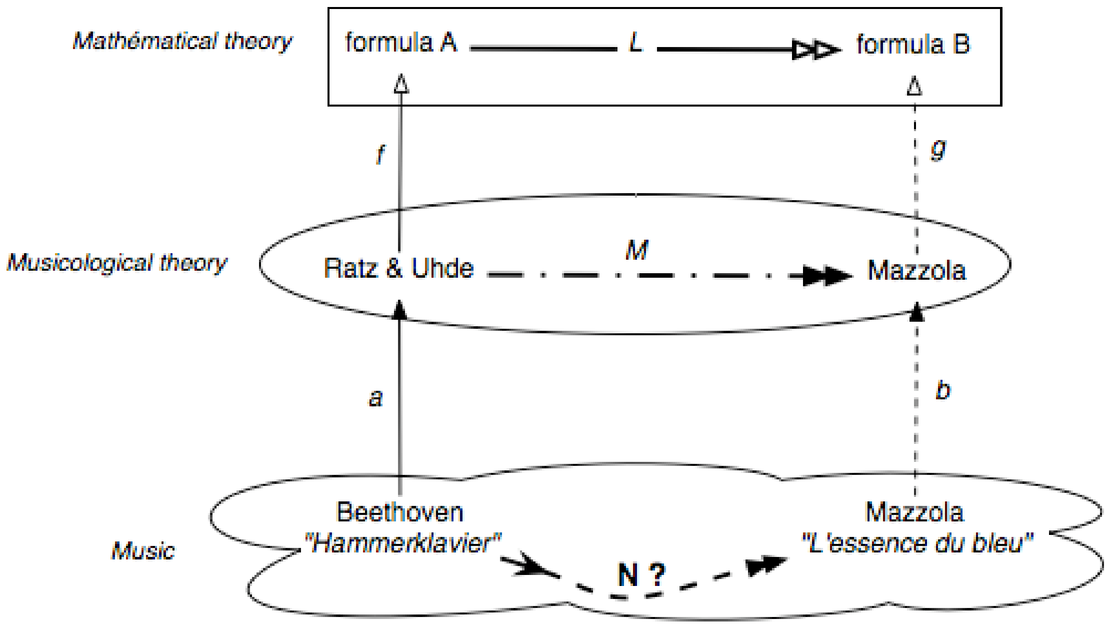
- mathematical formalization remains indifferent to the order of the harmonic sequences: she will consider, for example, that sequence VI→II→V→I (perfect cadenza) and sequence I→II→V→VI (broken cadenza) are mathematically equivalent in the same unit not ordered {I, II, V, VI}; this manner of seeing will astonish the musician…
- In the same way, mathematical formalization will consider that its “cadenza” {II-V} is equivalent to its “cadenza” {VII} since this last chord (B-D-F in major C), which can appear only in this tonality, is alone (among the other chords) to affirm major C. Again here, the musician will not recognize his music, his tonalities and his modulations: if, for the musician, sequence II→V is the gesture of a musical cadenza, on the other hand the simple statement of VII does not hold place of it since, quite to the contrary, this chord constitutes the prototype of the polymorphic pivot-chord [19], which is common to many tonalities.

In short, the musician does not recognize exactly his cadenzas and his modulations in the homonymous concepts of Mazzola, just as it could not recognize his own harmonic functions in eulerian formalization of the musical pleasure.

This torsion concerns a structural law; it does not come from one mathematician lazes or incompetence: cohesion of the musician experiment and coherence of mathematical formalization, musical logic and mathematical logic make clearly *two*. They can approach, enter in resonance, but they could not amalgamate, nor to even overlap. [20]

Let us give another example in the way in which the mathematical theorization tends to deform the musical neighborhoods that it undertakes to formalize.

To test its mathematical formalization of a musicological theory (by Ratz & Uhde) of the sonata *Hammerklavier* (Beethoven), G. Mazzola wonders whether it is possible to find an musical equivalent with a mathematical formula such as B, formula deductible (within the framework of its mathematical theory) from the formula A (which formalizes the sonata as theorized by the musicologists).



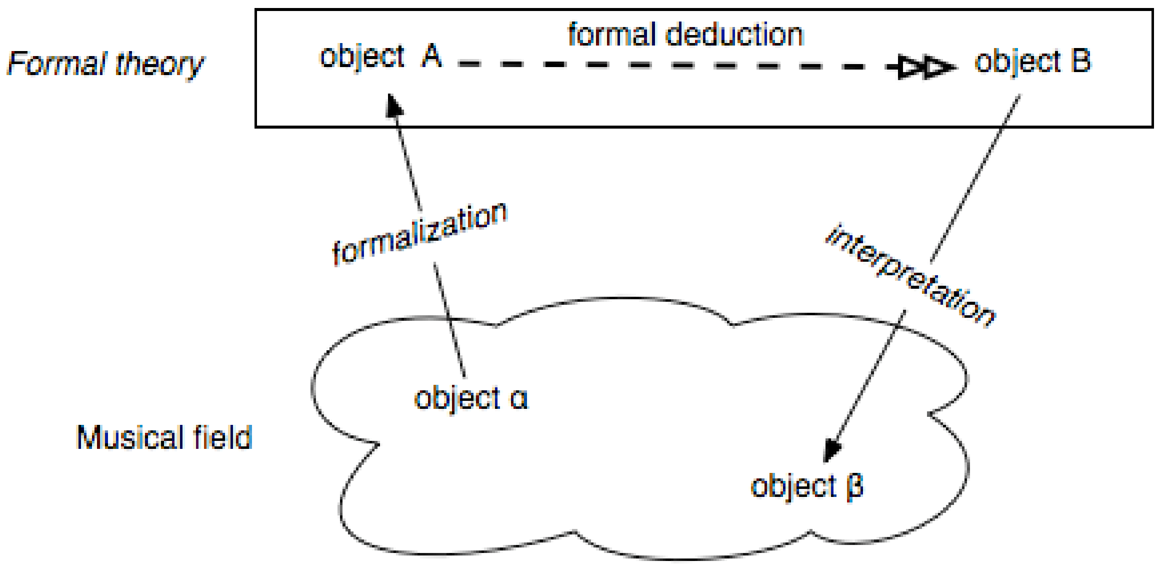
To carry out that, Mazzola composes a piece for piano (*L'essence du bleu*) whose musical analysis (arrow *b*), carried out according to the same musicological principles that for the sonata of Beethoven (arrow *a*), then mathematically formalized (arrow *g*) according to the same logic as that which was used for the analysis of the Beethoven sonata (arrow *f*), leads well to a related formalization B (arrows *L*) with starting formalization A.

It is understood that this device can ensure that there exists, in the theory of Ratz & Uhde, an arrow M such as the top rectangle commutates (i.e. such as g°M = L°f) since the new piece of music (*L'essence du bleu*) precisely was made up so that its analysis is quite related (by *M*) with the analysis (by Ratz & Uhde) of the Beethoven's sonata.

But the musician will address here to the mathematician an additional question: does there exist also a kind of arrow N - an arrow which is specifically musical (and either musicological or mathematical) - such as the rectangle of bottom (and thus also the complete rectangle) commutates i.e. such as b°N =M°a

(and $g°b°N = L°f°a$)? In other words, would this theoretical construction induce a bringing together of nature specifically musical between *Hammerklavier* and *L'essence du bleu*?
For the musician - who is the only one with being able to come to a conclusion about the properly musical existence of such a relation [21] -, such an "arrow" N does not exist in this precise case: to examine the two partitions (it is not the place, here, to do that…), it proves indeed that there is hardly musical relation between the sonata of Beethoven and the work composed ad hoc by Mazzola, which will not surprise us: it is not because a certain kind of musicological analysis can bring closer (arrow M) the analytical structures of two musical works that this is enough to musically connect these two works (just as two buildings could not be architecturally related as significant spaces because their plans would connect the same row of rooms or because one could count the same number of columns on their frontages…).
This raises how much such a mathematical theorization deforms the musical world: by founding formal neighborhoods which do not have musical counterpart, it brings closer musical objects remaining for the musician extremely distant, just as contrary it puts distant and separates what for the musician constitutes a neighborhood (see the families of harmonies built by Euler on the basis of his scale of the softness: they separate harmonies musically closed and bring closer musically distant harmonies).

Let us specify once again. This deformation of musical topology by mathematical formalization is not due to a negligence of the mathematician. It is an effect of structure, which is due to the point legitimating the logico-mathematical construction of a "model theory": mathematics seizes the field (which it will undertake to formalize) as discrete space of objects (they are their own neighborhood). Mathematical formalization will be thus formalization of the objects (here musical) but by no means of musical relation between these objects, relations which are *voluntarily* ignored [22]. The purpose of this formalization will be to build a new (theoretical) type of space where the new (mathematical) objects will be connected by deductive relationship, between musical field and mathematical theory being done by formalization and interpretation of the only (musical and mathematical) objects but by no means of their respective relationships.



The specifically musical relationship between α and β is deliberately ignored by formalization.

Technically known as, the theorization in question will thus not be *functorial*: formalization and interpretation will not be "functors" between two categories. [23]
Thus, if the specific interest of any formalization precisely holds with the contrasted relationship between a starting field formally seized like *discrete* (without immanent relationships) and a theoretical field where the objects will be connected by formal deductions, it goes without saying that the musical relationships (that the musician knows well but that the theory is unaware of) will appear to this musician as deformed and not reflected by the theoretical construction in question. It is to say that the musician reserves with regard to such a mathematical theorization will be inevitable.

A theory coordinating a sheaf of formalizations

To theorize mathematically the music engages a great diversity of formalizations which the mathematician will have to coordinate if he wants to build a theory of the music and not to accumulate a cluster of local operations.
Mazzola carries out this coordination within the framework which is offered to him by the grothendieckian topos theory [24]. Euler, of course, did not have such a preexistent framework, and its theorization of the music was useful to him precisely - *inter alia*… - to put the unit of mathematics of its time (then taken in a vast movement of diversification) proof against a single object (music)…
In both cases, a mathematical theory of the music is not satisfied to collect disparate formalizations and to deal with their mathematical unification. It is understood that such a requirement concerns a specifically mathematical command, and not musical at all. From where the following point.

A theory serving mathematics rather than music

Such mathematical theories, which aim at mathematics much more than music and which are the subjective business of mathematicians (the musician does not worry more unit of mathematics that the mathematician does worry to bring a new reading of such or such musical work) could not be of real use for the *working* musician.
The musician, craftsman of his art, will thus be interested hardly in these mathematical theories; quite simply it will not read them: it is not only that he could be made perplexed by such or such technical detail; it is more primarily that he does not need such a type of theory, as well in its practice as in its possible concern of theorizing the music this time as a musician (one will examine further the specific manner whose musician will be able to seize mathematics to theorize the music).

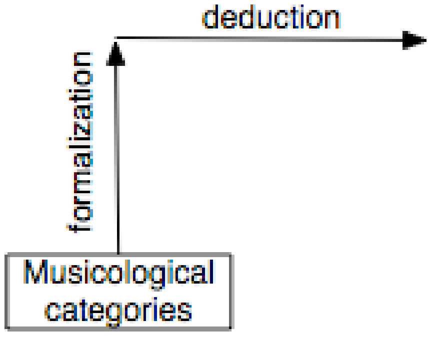
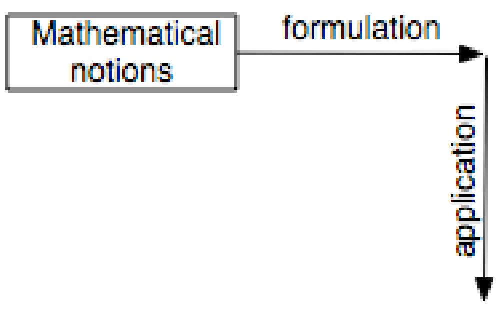
A theory producing of new knowledge on the music

This implies by no means that such a mathematical theory of the music remains vain for the musician, at least for the pensive musician [25]. For example the formalization of Mazzola leads to this remarkable result: the theory of the counterpoint by Fux and the theory of the harmony by Riemann prove narrowly related by this theory according to the geometry of intervals which them framework, when, however, these two theories (of Fux and Riemann) remain separated by the chronology (respectively XVIII° and XIX° centuries) and by the practice of the musicians (in music, counterpoint and harmony give place to disjoined lessons, without theoretical unification [26]).
Thus this mathematical theory reveals structural properties, up to that point unperceived of the musician and of the musicologist. It is to say that this theory makes it possible to extend the knowledge on the music even if it does not make it possible to invent as regards musicians practices.

It is for this reason that this mathematical type of theory will interest the musicologists rather than the musicians if it is true that all and sundry are distinguished as follows: the musicologists constitute themselves around knowledge in externality on a music conceived like object already there, when the musicians proceed of knowledge in interiority of a music which they make.

II. *MUSICOLOGIST* MANNER OF THEORIZING THE MUSIC

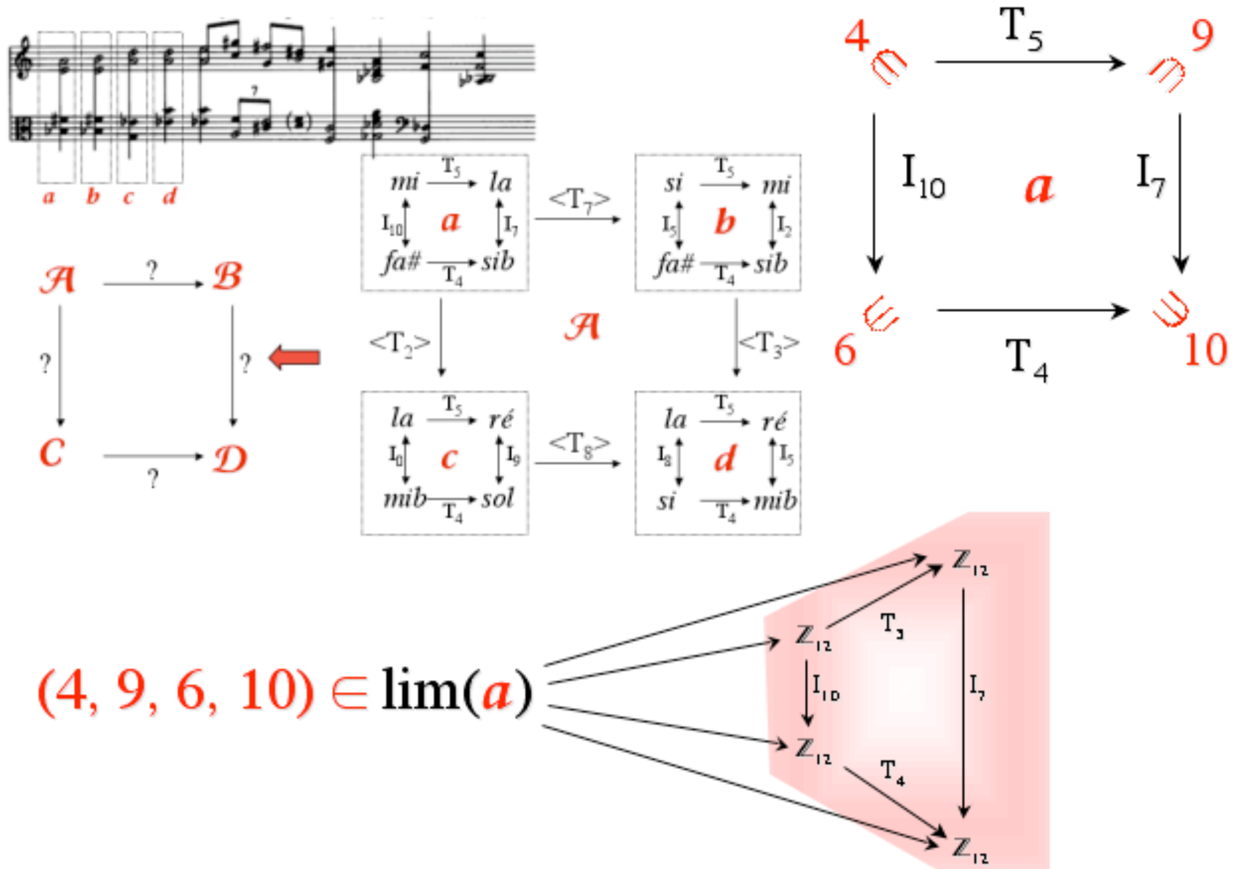
The musicological manner to theorize the music with mathematics will operate contrary to the mathematician manner: it will leave this time a mathematical theory which preexists to apply it to such or such musicological question.
One can present the contrast of these two dynamics in the following way:

Mathematician manner :	*Musicological* manner :
	
It mathematically formalizes musicological	It builds, starting from mathematical concepts (which

categories (which it takes in the existing musicological corpus) to deduce in properly mathematical space from its objects.	it takes in the existing mathematical corpus), an original formulation likely to be applied in the specifically musicological space of its categories.

Altogether, the musicological manner of theorizing the music with mathematics consists in building a "mathematical model" for a musicologically given problem: if mathematician formalization can be conceived like a "mathematization" of the music, the musicological manner will consist rather of a mathematical "modelization" of musicology [27]. Thus this last manner privileges, in mathematics, its capacity of calculation rather than the power of its concepts.
This kind of musicological theory is committed thus to what is called "a computational musicology". In *mamuphi*, the carrier more succeeded of this orientation is Moreno Andreatta. [28]

The work of this musicology is carried out especially in the pure algebra (primarily the group theory) but an important part is based from now on a modelization in term of topos. For example, this relates to what the *music theory*, since David Lewin, calls the "transformational" approach of the pitchs networks. About what is it?
It is initially a question of segmenting a score in pitchs groups - say in "chords"… - connected to each other (*transformational network*) by musical operations of *transposition* and *inversion* in kind to produce a total recovery of the concerned score. It results from this the constitution of an abstract space: that of the transformations in the course of the time (*transformational progression*) of the constitutive groups of this network.
This way of insisting less on the particular nature gathered pitchs that on the structure of the transformations to which these groups give place organizes a musicological matter which lends itself then quite naturally to modelization of a categorical type privileging in the same way the relations between objects.



(modelization of a musicological analysis by D. Lewin of Schoenberg's op. 11 n°2)
More precisely, the musicologist, anxious to enumerate and classify these musical structures ("Klumpenhouwer networks"), will model them in a toposic way (see the *limit* for the last diagram). The result, once implemented by means of computer[29], will be able to release the good strategies of analysis concerning the networks working in such or such score. Thus musicological modelization by the topos will lead directly to a musicological analysis computer-assisted.

Also let us mention a feed-back effect on mathematics of this musicology: certain questions, addressed by this formalization to mathematics, will be able to cause new mathematical problems. It is there what Moreno Andreatta likes to call a "*mathemusical*" problem: a musicological problem addressed to mathematics which is such as its formalization causes new theorems opening then with new musicological applications. [30]

III. *MUSICIAN* MANNER OF THEORIZING THE MUSIC

There remains a third manner, extremely different, to theorize the music in the light of mathematics: that of the musician - i.e. of course of the *working musician* (there is not an other type of musician only this one!).

The musician is distinguished from the two preceding orientations because its theorization will not aim at producing a "theory" like such: its theorization will concern rather what Louis Althusser had called a "theoretical practice", i.e. an intervention whose stake is not any more the constitution of a theoretical system, stable and transmissible ("a theory"), but the release of an idea musician of the music. [31] For this reason, one will be able to say that the properly musician theorization is a *ideation*. [32]

Methodologically, the recourse to mathematics to theorize the music thus will be carried out under the sign of what one will call, following Gaston Bachelard, an *experimentation* of the thought: it will be a question for the musician at the same time of formalizing and of interpreting the musical categories and the mathematical concepts in kind to put its discursive thought proof against mathematical coherence.

Let us give for that an example, which I will borrow this time from my own work.

To theorize a *Music*-world like topos…

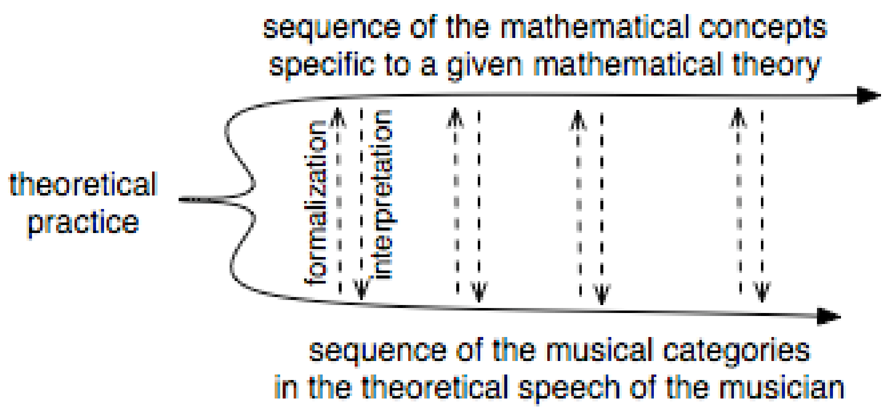
Let us suppose that a pensive musician of today feels the need to theorize how the music can form a particular world; for such a project it does not miss the good reasons: for example its desire to be opposed to this new practice, for him hateful, to put the expression "the musics" instead of the ancestral expression of the musicians: "music". [33]

This musician would like to thus support in thought that there exists well a world of the music (and not only one area which one can roughly delimit in a general universe) and only one, and that this world, though internally diversified (like any world!), remains connected (all that occurs in some place from this world relates potentially to any other place). In short, the musician would like to be able to say music what Alain Connes says of mathematics: "*there is only one mathematical world*" [34] and "*this mathematical world is connected*" [35].

But for that, how to proceed, how to found such an musician idea of one and only one musical world?

The musician, then, will be able to turn to mathematics while saying himself [36]: "the grothendieckian concept of topos provides a strong contemporary mathematical idea of what is a world; thus let us put our musician idea of a musical world proof against this mathematical idea of topos."

The musician will then start a theoretical practice which will simultaneously explore the double sequence of the mathematical concepts and the musical categories according to the following movement:



In our example (how to theorize, in the light of grothendieckian mathematics of topos, the music like a world?), this experimentation [37] will lead the musician to the following tasks:

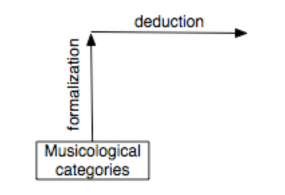
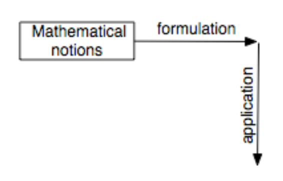
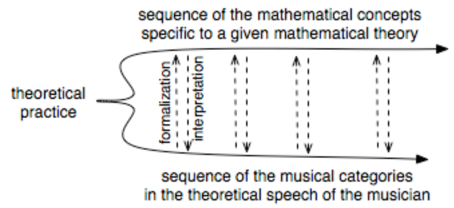
1. to formalize a piece of music like a *sheaf* of executions of its score;
2. to formalize the library of the scores of music like a *site* of its *quodlibets*;
3. to formalize the world of the music like a *category* of the works extracted from this library;
4. to formalize the world of the music like a *topos* of all these works-sheaves;

5. to draw, in the course of work, all useful and relevant conclusions concerning the musical objects and their relations.

It is understood that this musician experimentation of the mathematical concepts will hardly interest the mathematicians, since the effects to wait of such a theorization will remain intrinsically musical.
This experimentation will not more interest the musicologists who will not recognize there the procedures regulating their "objective" production of knowledge. [38]
One perceives here a difficulty, specific to confrontations which animate the *mamuphi* meetings: it does not go from oneself that the mathematician, musicological and musician theorizations can be interested between them. Thus the specific challenge of the *mamuphi* project is precisely to put in resonance these distinct theoricities, as well objectively as subjectively (one indicated it besides: it is exactly here that the shade of philosophy is required).

ON THE WHOLE…

Let us summarize our three main trends.

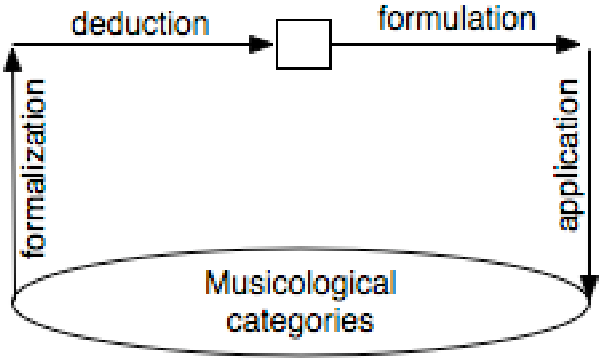
	Mathematization or mathematician *formalization*	**Modelization** or musicological *application*	**Experimentation** or musician *theoretical practice*
			
Stakes of this theorization:	to make mathematics while widening the power of mathematics and consolidating their unit	to produce, in objectifying externality, new knowledge on the music	To deepen, in subjectifying interiority, musical knowledge
Result of this theorization:	a (mathematical) theory of the music	a (musicological) theory of the music	an (musician) idea of the music
The music is:	an indirect origin (via musicology)	an indirect target (via musicology)	a sensible space of thought
Mathematics is:	a target	an origin	a conceptual space of thought
Mathematics concerned takes the form of:	*theories*	*formulas & equations*	*concepts*
The music-mathematics ratios privilege:	formalizations	interpretations	resonances, therefore *mathems*

Relations between these three theorizations

Even if each one understands where the personal preferences of the author (musician) of this article go, it is however clear that each orientation distinguished here has its own coherence and that there does not exist position in overhang [39] which would authorize to treat on a hierarchical basis our three theorizations.
However the preceding table states that the 3 of our orientations can be deducted, in three manners, in 2+1.

Complementarities mathematicians/musicologists

Firstly the mathematician and musicological positions arise in our table like dual and/or complementary. This complementarity authorizes a new manner, this time *mixed*, to theorize the music with mathematics, manner which connects *mathematization* and *application* [40] :



Complicities mathematicians/musicians

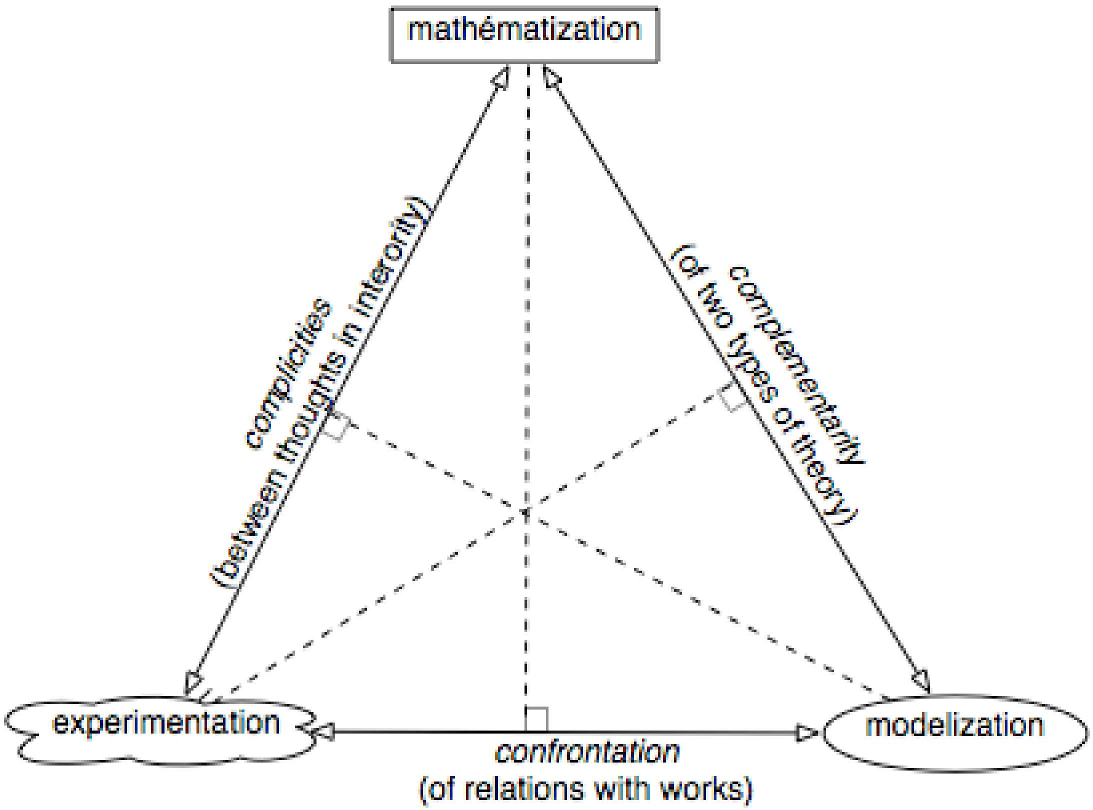
Secondly, one can then notice that mathematician and musician orientations cause narrower complicities between thoughts in interiority than does not cause a musicological practice of very technicized modelization, practical privileging the computing power of mathematics and exteriorizing the "objective" dimension of the music.

Confrontations musicians/musicologists

Thirdly, musicological and musicians theoricities meet around the scores since they give the same direct attention to them. This will maintain between them what one will call here, with a nice euphemism, a healthy competition…

General geometry

The musician experimentation being "orthogonal" with the complementarity of the mathematical and musicological theories, - as musicological modelization is orthogonal with complicities between thoughts in interiority (mathematician and musician) and as the mathematization is orthogonal with musicians/musicologists confrontations relating to the scores -, the *mamuphi* geometry which proceeds of these relations could be thus drawn:



A counterpoint…

On the whole, and according to a musical metaphor, the relations between our three theoricities give to the polyphonic development of *mamuphi* the pace of a counterpoint.
As the musicians know it well, they are the dissonances - not the consonances - which make the music, and these dissonances, at least since Schoenberg, do not need more to be solved to remain musical.

Thus, the musician will be able to await the best of these *mamuphi* dissonances and orthogonalities: it was here necessary for him to make them clearly hear, so to restore them according to a *mezzo-forte* (*mf*) rather than a *pianissimo* (*pp*)…

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[1] http://smf.emath.fr/Publications/Gazette

[2] http://smf.emath.fr/Publications/Gazette/2009/119/smf_gazette_119_35-49.pdf

[3] Compositor (*École normale supérieure*/*Ircam*)
http://www.entretemps.asso.fr/Nicolas

[4] *Société mathématique de France*: http://smf.emath.fr

[5] http://smf.emath.fr/VieSociete/JourneeAnnuelle/2008
http://smf.emath.fr/VieSociete/JourneeAnnuelle/2008/Resumes.html

[6] *Gazette*, July 2008, n°117 : http://smf.emath.fr/Publications/Gazette/2008/117
http://smf.emath.fr/Publications/Gazette/2008/117/smf_gazette_117_35-47.pdf

[7] The opportunity to start *mamuphi* was provided by an initiative (at the end of 1999) of the EMS (*European Mathematical Society*), which, within the framework of its *Diderot forum* (http://emis.math.ecnu.edu.cn/etc/diderot4.html), had chosen "logic" as the issue to be debated in workshops with Ircam.

[8] http://www.entretemps.asso.fr/maths

[9] One will return to the two *mamuphi*'s inaugural books:
- *Mathematics and Music (A Diderot Mathematical Forum)* ; ed. G. Assayag, H.G. Feichtinger, J.F. Rodrigues ; Springer-Verlag, 2002 - http://www.maa.org/reviews/mathmusic.html
- *Penser la musique avec les mathématiques ?*; éd. G. Assayag, G. Mazzola, F. Nicolas ; Delatour, 2006 - http://www.ircam.fr/598.html?&tx_ircamboutique_pi1[showUid]=172&cHash=bb50400732

[10] http://www.entretemps.asso.fr/Grothendieck
http://www.grothendieckcircle.org

[11] i.e. the fifth of the twelve "great ideas" which it releases in "*Récoltes et Semailles*" (2.8)
http://www.math.jussieu.fr/~leila/grothendieckcircle/RetS.pdf

[12] My own reference books on the matter are *Topoi. The Categorial Analysis of Logic* of R. Goldblatt (North-Holland, 1984) and *Sheaves in Geometry and Logic. A First Introduction to Topos Theory* of Saunders Mac Lane & Ieke Moerdijk (Springer-Verlag, 1992)

[13] See its two reference books:
- *The Topos of Musi*c, Birkhaüser, Basel, 2002
- *La vérité du beau dans la musique*, Delatour, Paris, 2007

[14] Here, one will not present systematically this vast mathematical theory. It is here only a question of reading this theory as a musician, i.e. a reading remaining more attached to delimit its matter and to distinguish its mathematician subjectivity than to explore the properly mathematical depth of it.

[15] Musicology was invented only during the XIX° century, under the double influence of the German historicism and French positivism…

[16] Let us specify that this "diagram" (as those which will follow) only gives one indication of an guiding idea. Thus it has only illustrative value: the points and arrows which appear here have relationships only metaphorical with the objects, morphisms and functors of the category theory.

[17] The philosopher Charles Alunni, co-organiser of the *mamuphi* seminar, proposes to regard it as one *tra(ns)duction*.

[18] *Gazette*, July 2008, n°117 (op. cit.)

[19] Technically, this chord VII concerns a diminished seventh. This chord materialized tonal uncertainty by avoiding precisely any cadential logic. Thus the less (musically) "cadential" chord corresponds, in mathematical formalization, with the more (mathematically) "cadential" chord…

[20] As one indicated, it is at this place that the shade of philosophy is necessary.
In few words, music and mathematics make radically *two*, without possibility - other that (neo)-positivist or scientistic - of uniting them. That is due to the irreducible singularity of the musical work of art. As always, it will go from there, in this precise place, of sharing between axiomatic:

- either one supports that "there are works of art" (Hegel), that these works are in art the true subjects; in this case mathematics could only be unaware of this specificity to seize work-subjects only by their ontic dimension (that of simple "pieces of music");
- either one supports that there is not meaning to distinguish between pieces of music and musical works, that there is no place in music for a figure of "subject", for something like a "musical subject"; in this case mathematics will be able "to seek" to formalize the music completely as it can legitimately seek to completely formalize the movement of planets, the reproduction of the ants or the food preferences of the human animals. But such does a project (to reinstall the music under supervision of mathematics) constitute for mathematics a *real* ambition? Without coming to a same conclusion about the question of works, Euler knew, in all the cases, to avoid such a covetousness and to respect the autonomy of the world of the music, without to lose there (quite to the contrary!) the power of thought suitable for mathematics.

[21] We point out Euler: "*In music, as in all the fine arts in general, it is necessary to be aligned on the opinion of those which have at the same time an excellent taste and much of judgment. Consequently it is necessary to hold account only of opinion of people which, having received nature a delicate ear, perceives with accuracy all that this body transmits to them, and is able to judge some in a healthy way.*" It is thus a question "*of consulting the metaphysicians* [= here musicians] *that this search relates to*" (*Tentamen*…, chap. II)

[22] From this point of view, the particular case where mathematics formalizes an preexistent "empirical" theory (here musicological one) - thus a field this time not "discrete" since equipped with internal relations (of proximity, distance, sequence, etc) and thus with neighbourhoods non reduced to only one point - constitutes only one alternative since formalization and interpretation will there continue to relate to only objects, and by no means to morphisms. The theorization thus considered will not produce more functor between musicological and mathematical theories: the musicological theory being used as starting field remains too empirical then to be truly formalizable in a mathematical category.

[23] A fortiori, one cannot have adjunction between musical field and mathematical theory. An important aspect of the *mamuphi* internal debates relates to this precise point…

[24] Let us note its systematic reinterpretation of categorial morphisms like addresses (familiar with theoretical informatic), x→y being rewrited like x@y..

[25] The musician tends to becoming pensive "when the music stops" (Th. Reik), the musician finding itself then temporarily vacant out of musical world. It is the moment when he is naturally led to reflect on what arrived to him, to charge its musical experiment to encourage itself to continue its to and from (in/out Music-world).
Like the mathematicians, the musicians are regularly subjected to nihilist temptation: the temptation of "What good is it? ", "in vain" (Nietzsche). That the abandonment of their cause often takes the form not from a desertion but from a academization does not withdraw anything with the fact that it is indeed a subjective resignation.

[26] The unification is carried out only practically, for example by the school exercise of the choral and of the fugue…

[27] Let us recall that "model" gets busy here contrary to meaning that this word has in (logico-mathematical) "model theory". In "model theory", the word "model" indicates the original to copy; in "mathematical modelization", the same word "model" indicates the reduced model, the model to be interpreted. For a discussion of the philosophical meaning of this (neo-positivist) inversion, one will return to the book of A. Badiou: *The Concept of Model*, transl. by Zachery Luke Fraser & Tzuchien Tho (Melbourne: repress, 2007).

[28] Let us indicate that this computational musicology finds a natural prolongation in a particular seminar, related with *mamuphi*, which is held in Ircam under the name *MaMuX*: http://www.ircam.fr/equipes/repmus/mamux
One will find in the *Journal of Mathematics and Music* many contributions to this new type of musicology: http://www.tandf.co.uk/journals/titles/17459737.asp

[29] To see *From has Categorical Point of View: K-Nets ace Limit Denotators* (G. Mazzola and Mr. Andreatta, *Prospects of New Music*, 44-2, 2006) and, more generally, works of the team *Musical representations* in Ircam: http://recherche.ircam.fr/equipes/repmus

[30] It would thus, in my opinion, be a question rather of a “mathemusicological” problem…

[31] This *musician* idea on the music is distinguished, of course, of the *musical* idea: that which, in the course of the work, takes the shape of a musical object, for example of a theme.

[32] I call “musical intellectuality” this musician ideation.
We thus do not mislead on the theoretical work of Rameau, this pioneer of musical intellectuality. Its evolution stresses that, since the departure, it was a question for him of intervening theoretically for the benefit of a certain (harmonic…) idea of music, badly established at its time; its “theory” was thus a (theoretical) manner of pleading its cause of “harmonic” musician rather than melodic one by giving to this “theory” strong bases, rooted in the rationality (in particular Cartesian) of its time.

[33] Let us say that it is, here, about a concern specifically musician for the unit of the music. This concern is very equivalent to the eulerian concern of preserving the unit of mathematics beyond the beneficial diversity of its practices.

[34] *A View of Mathematics*: http://www.alainconnes.org/docs/maths.ps

[35] *Les déchiffreurs*, p. 14, Belin, 2008

[36] The musician will be thorough with this operation by the book of Alain Badiou (*Logics of Worlds: Being and Event, Volume 2*, transl. by A. Toscano; New York: Continuum, 2008) since this book supports that the philosophical concept of world must be established today under condition of the mathematical category of topos.

[37] For more details, one will be able to refer to a first presentation of this *work in progress*: http://www.entretemps.asso.fr/Nicolas/2008/Faisceaux.htm

[38] So that a musicologist can be interested in a “idea musician”, it is necessary for him initially to vitrify it in a “musicological object” …

[39] Philosophy does not concern more from a Sirius point of view…

[40] This manner operates very directly in *mamuphi*: on a side the works of Mazzola, which worry to informatically implement its theory, stress the computational repercussions of its mathematical theory and, on another side, musicological work of Andreatta roots narrowly in the mazzolian theory of the music.