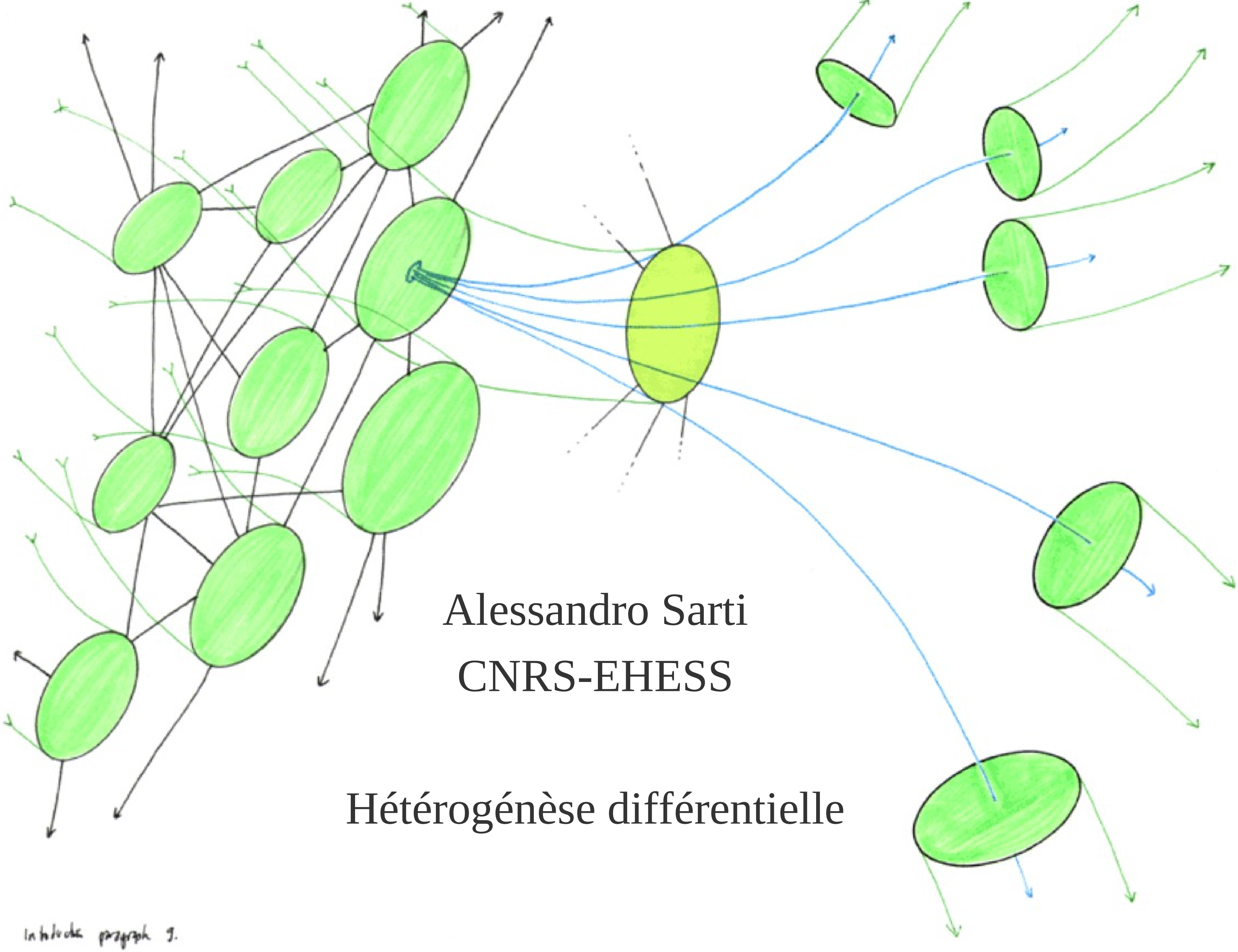
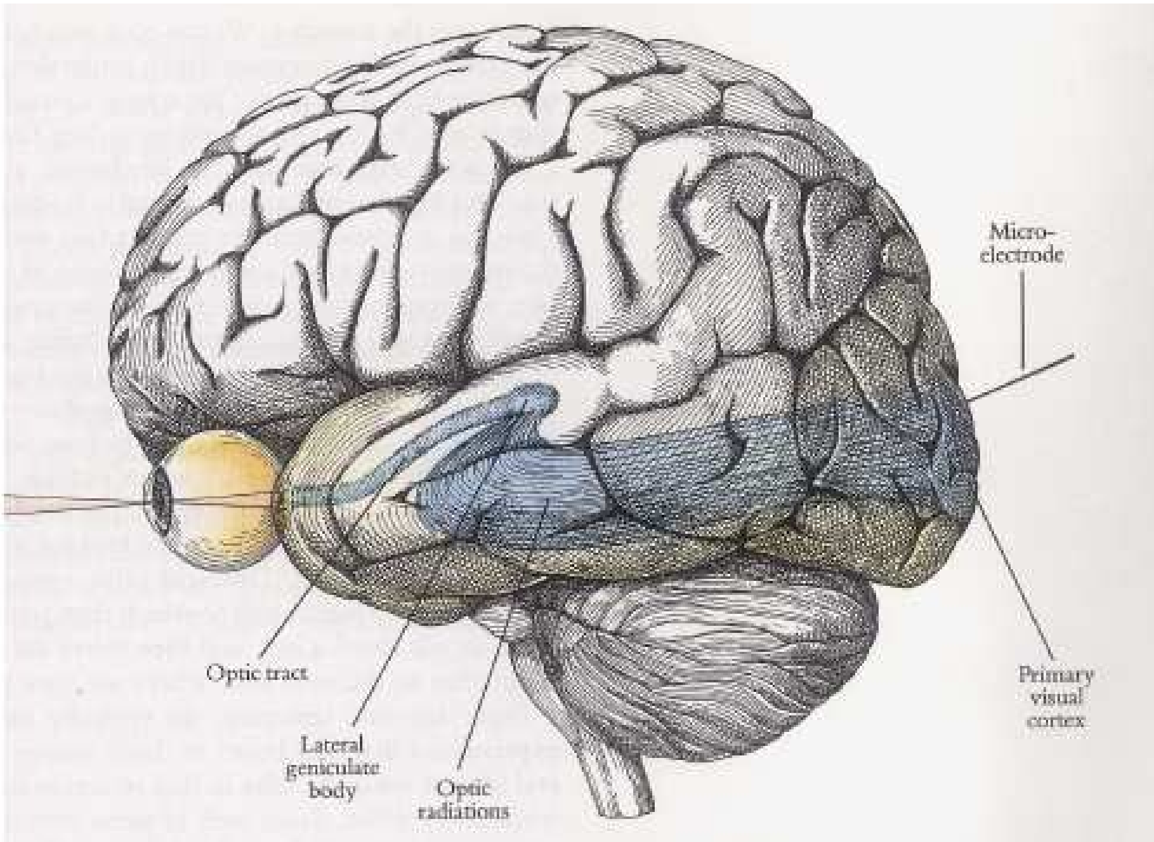


Alessandro Sarti

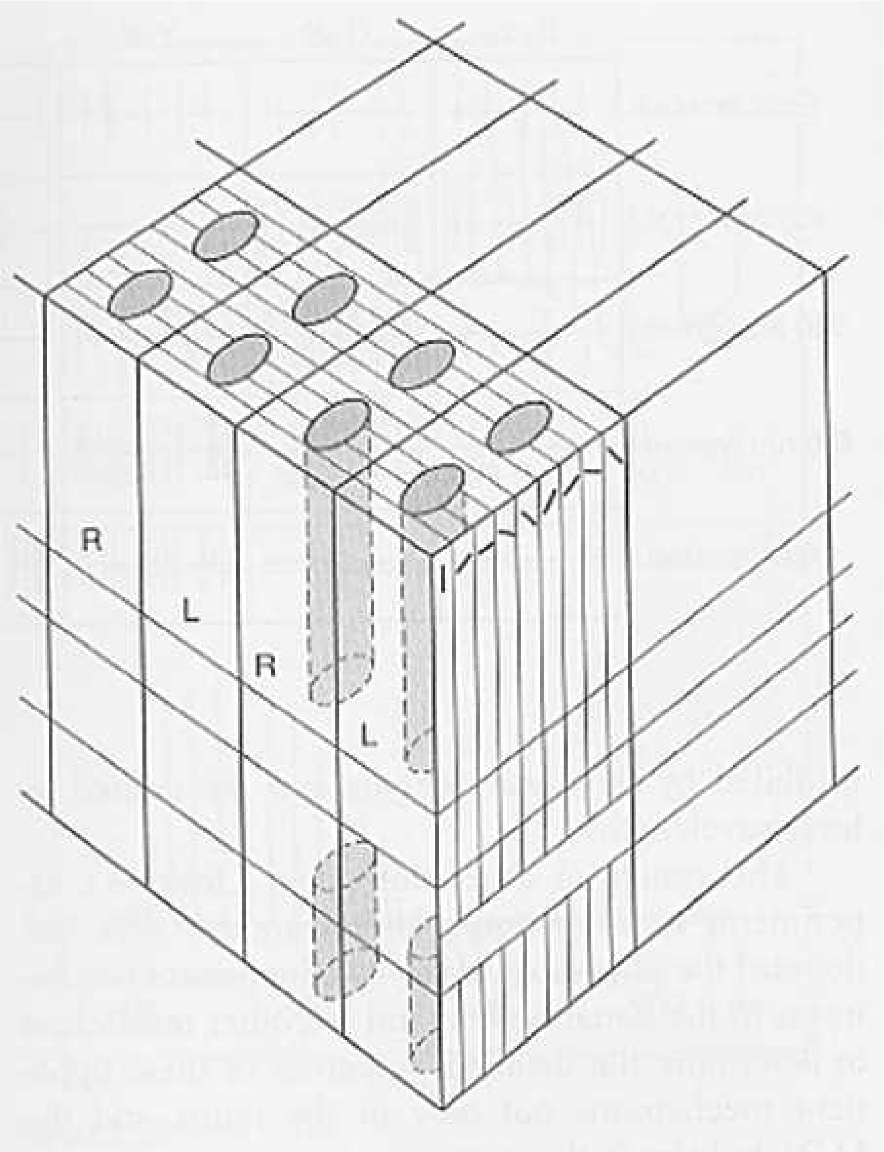
CNRS-EHESS

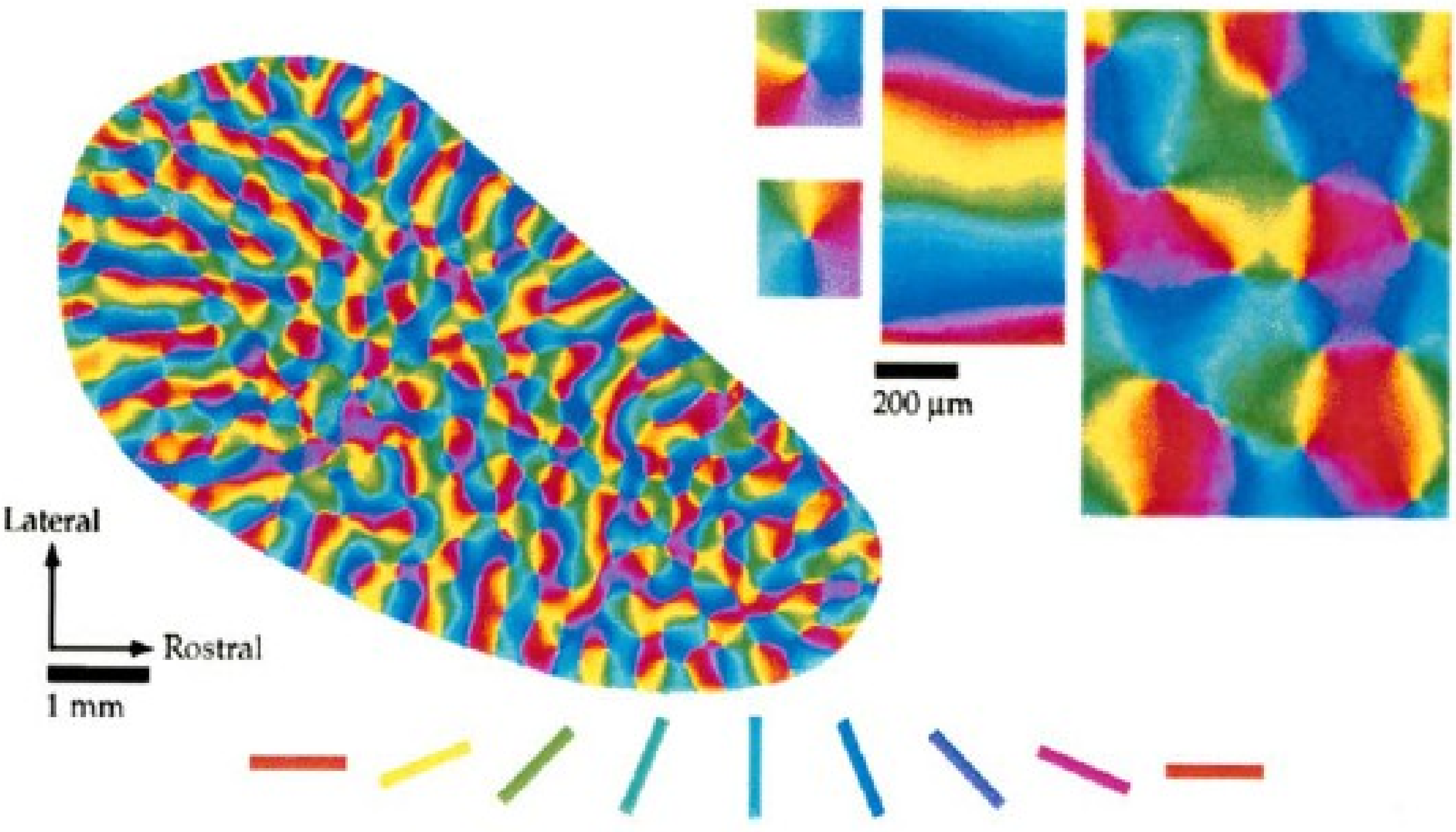
Hétérogénèse différentielle

The visual pathway



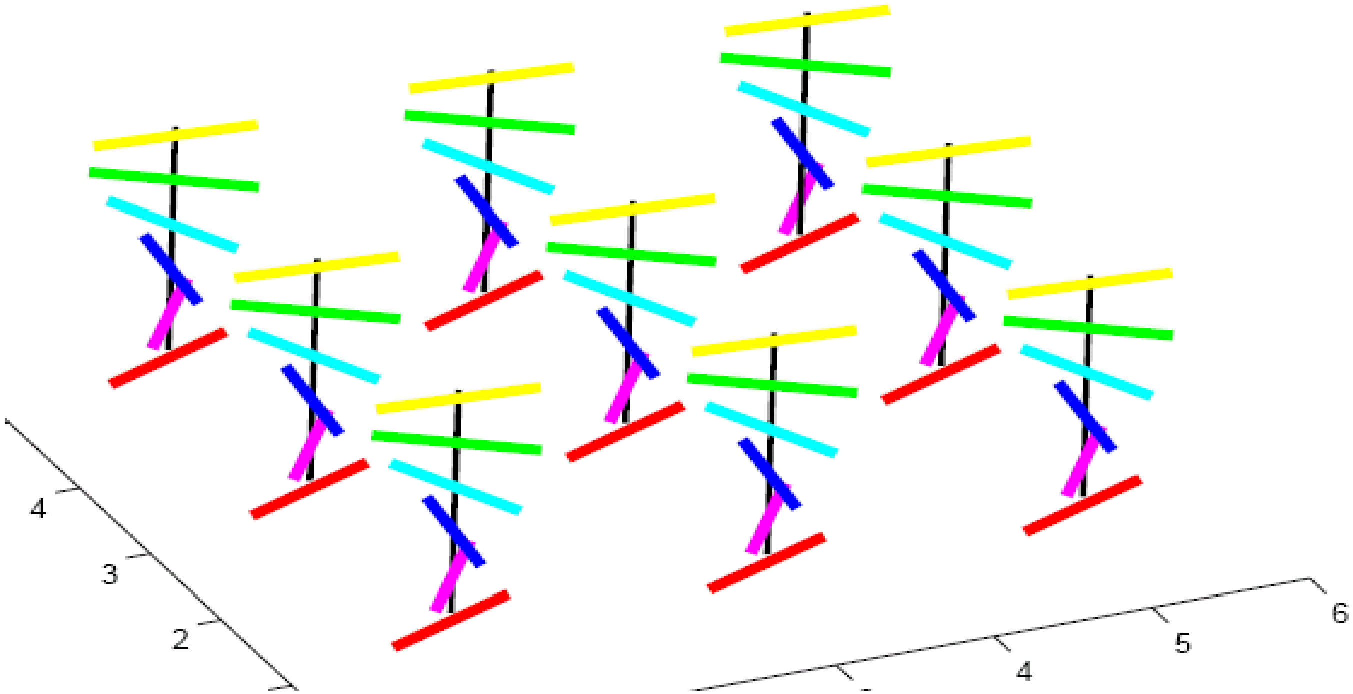
The hypercolumnar module





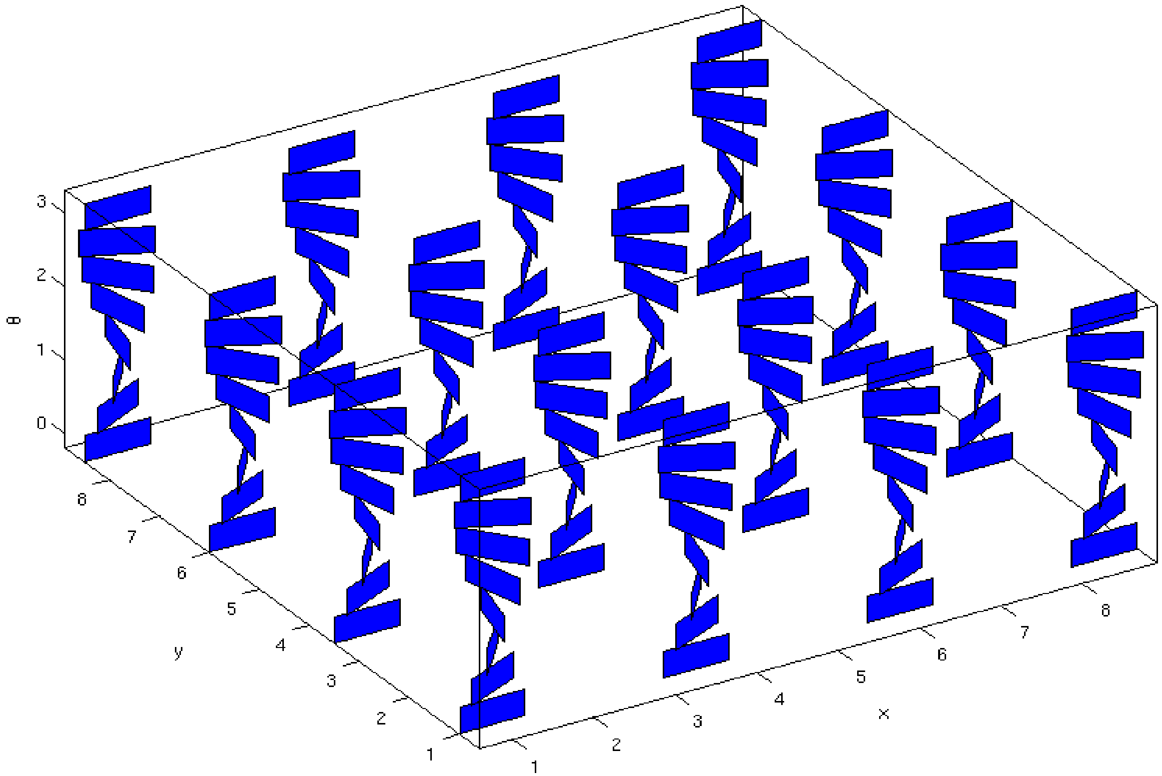
Lateral
Rostral
1 mm
200 µm

The Lie group of rotation-translation SE(2)



$$G = \left\{ (r, z) = \begin{pmatrix} e^{ir} & z \\ 0 & 1 \end{pmatrix} \middle| r \in \mathbb{R}, z \in \mathbb{C} \right\}$$

Infinitesimal transformation and the Lie algebra

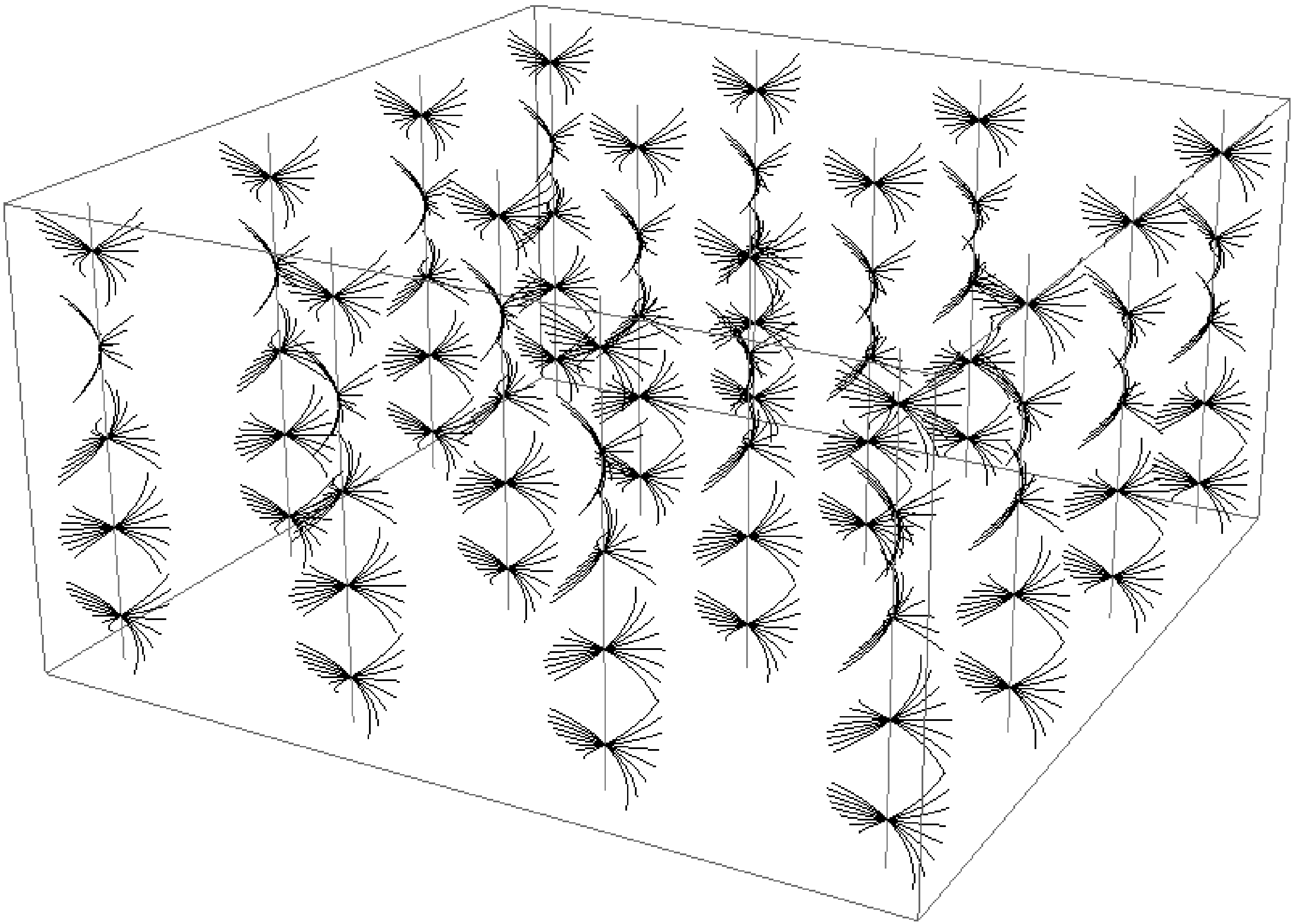


$$\vec{X}_1 = (\cos(\theta), \sin(\theta), 0)$$

$$\vec{X}_2 = (0, 0, 1)$$

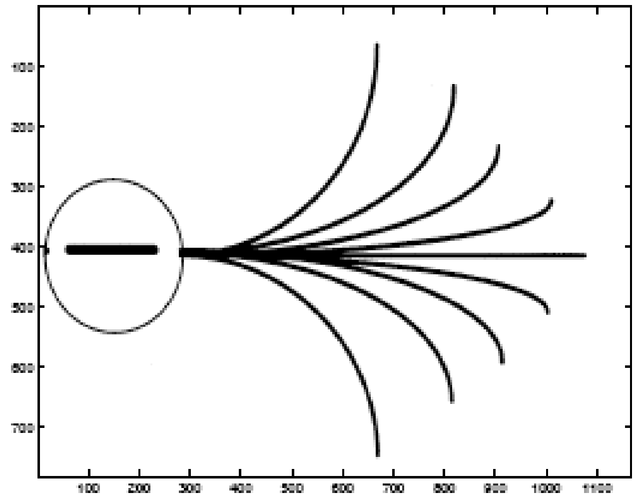
G.Citti, A.Sarti 2006

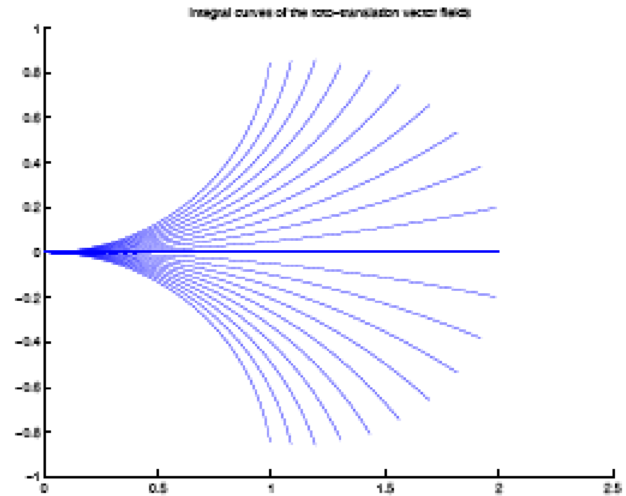
Integral curves and connectivity



$$\gamma'(t) = \vec{X}_1(\gamma) + k\vec{X}_2(\gamma), \quad \gamma(0) = (x, y, \theta)$$

Association Fields and Integral Curves of Tangent Vectors



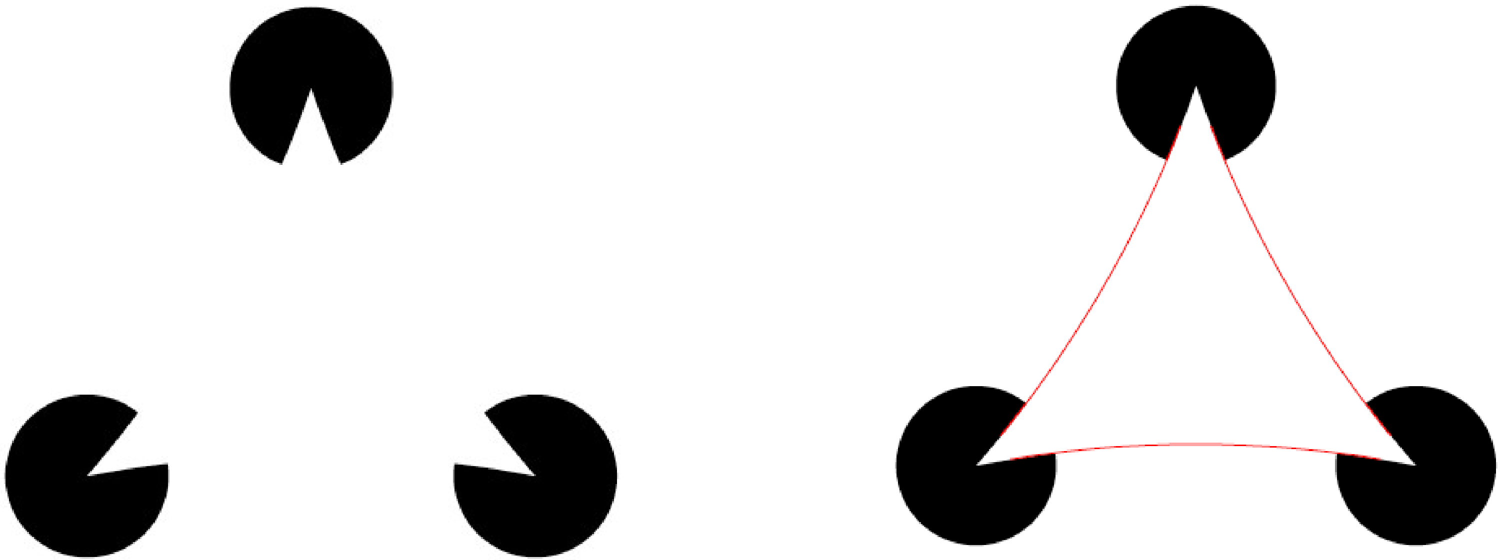


$$\gamma'(t) = (\gamma_1 \vec{X}_1 + \gamma_2 \vec{X}_2) \quad \gamma(0) = (x, y, \theta)$$

$$\vec{X}_1 = (\cos(\theta), \sin(\theta), 0)$$

$$\vec{X}_2 = (0, 0, 1)$$

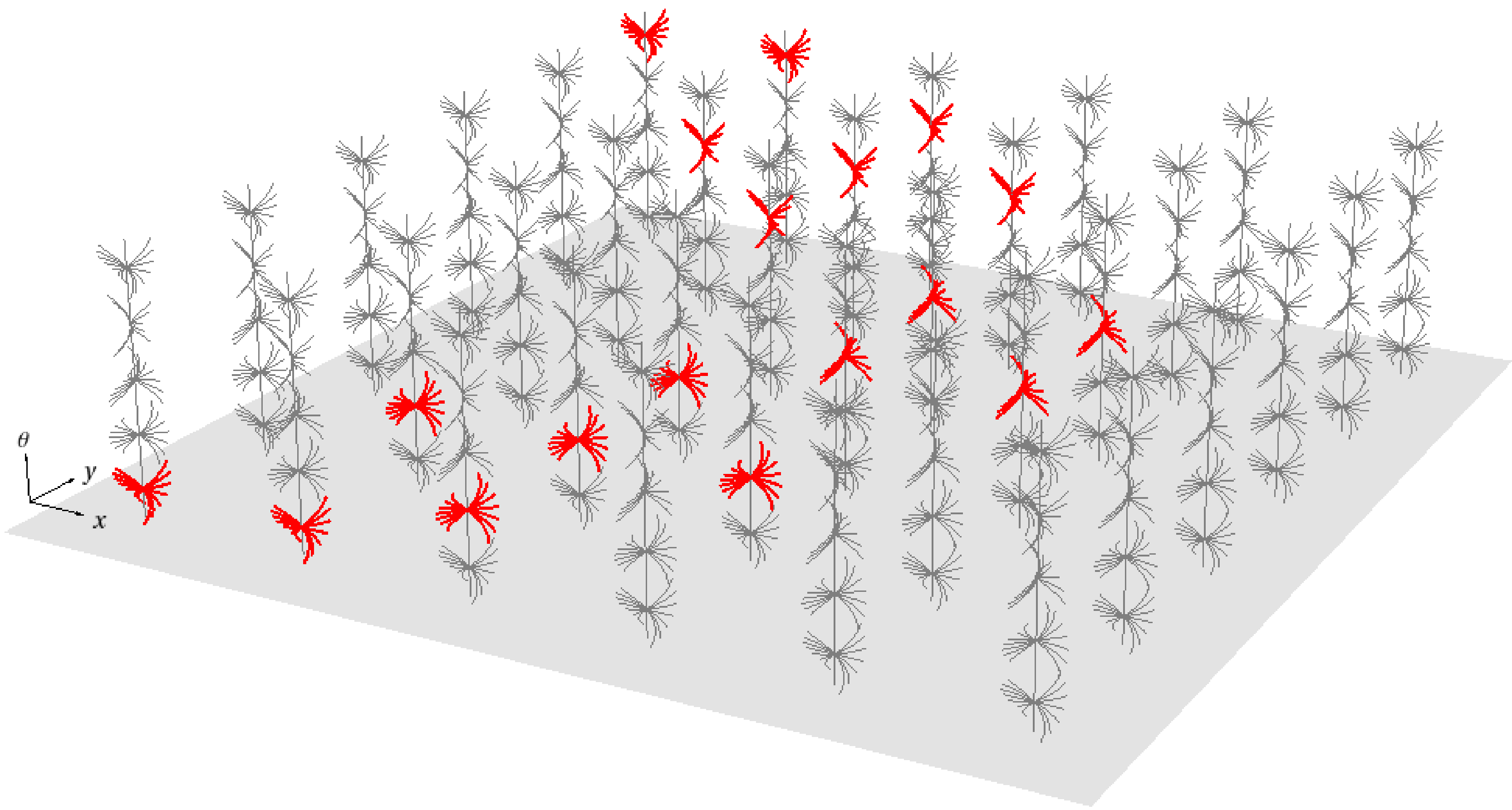
Contour completion with geodesic curves



On the constitution of perceptual units

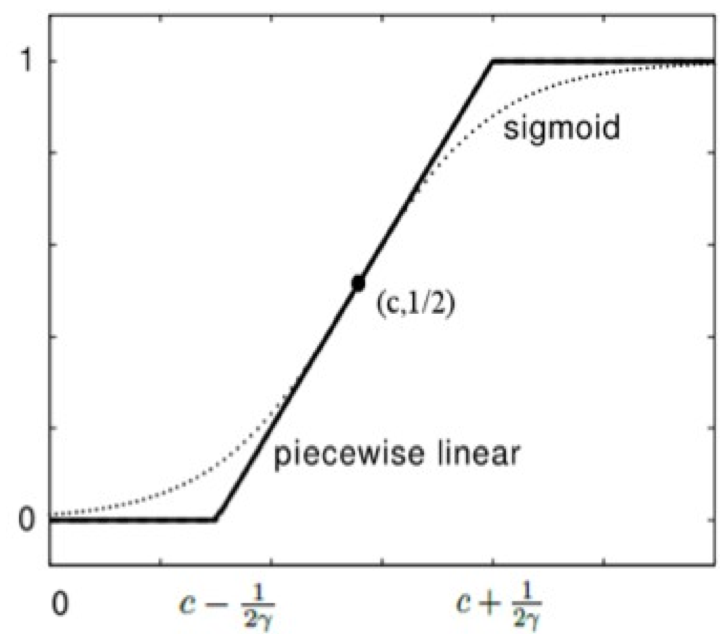
Constitution of perceptuel units

The excited network

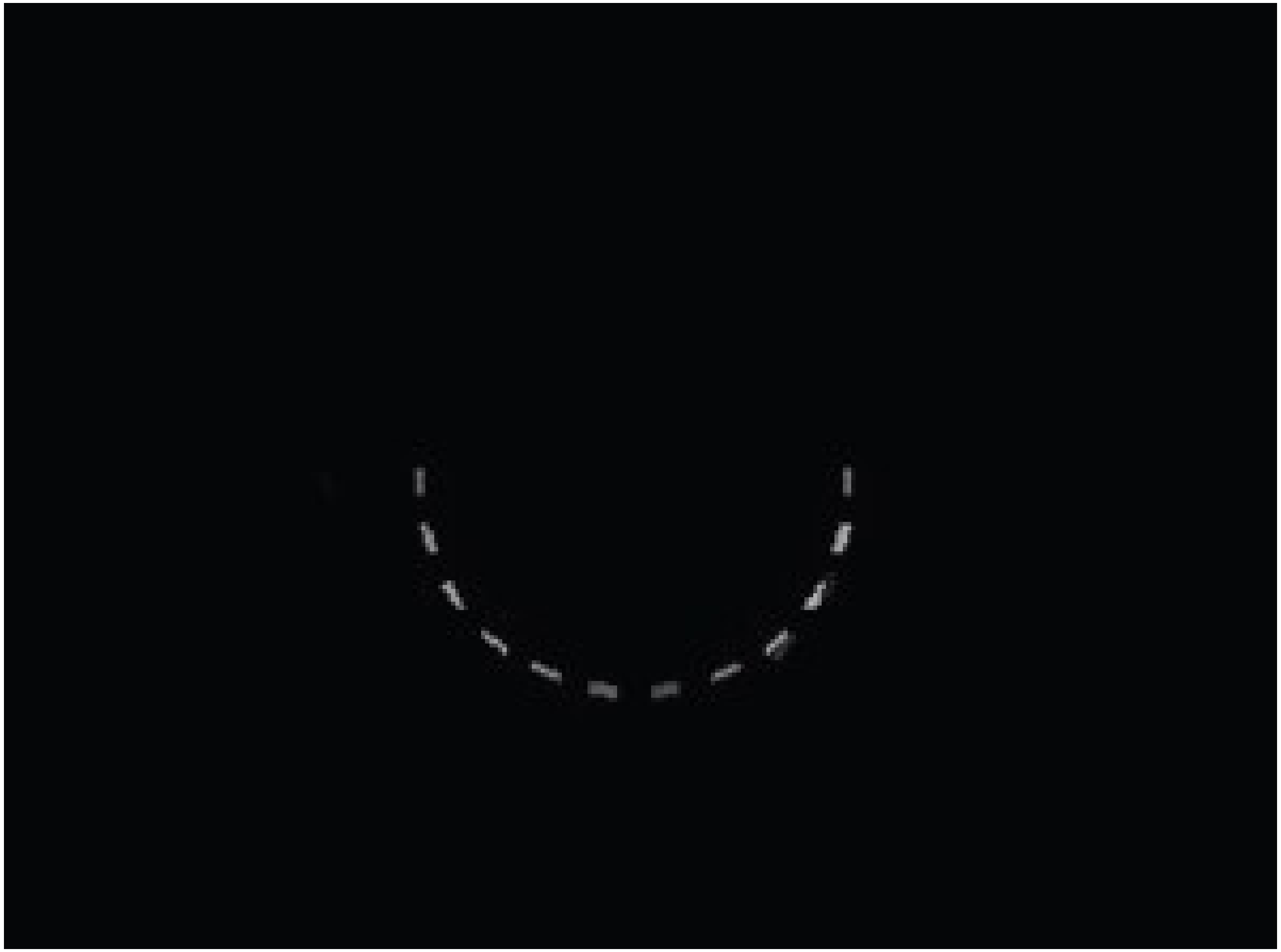


Ermentraut-Cowan mean field equation of neural activity

$$\frac{\mathrm{d}a(\xi,t)}{\mathrm{d}t} = -\alpha a(\xi,t) + \sigma\Big(\int \mu\omega(\xi,\xi')a(\xi',t)d\xi' + h(\xi,t)\Big) \quad \text{in } \mathcal{M}$$



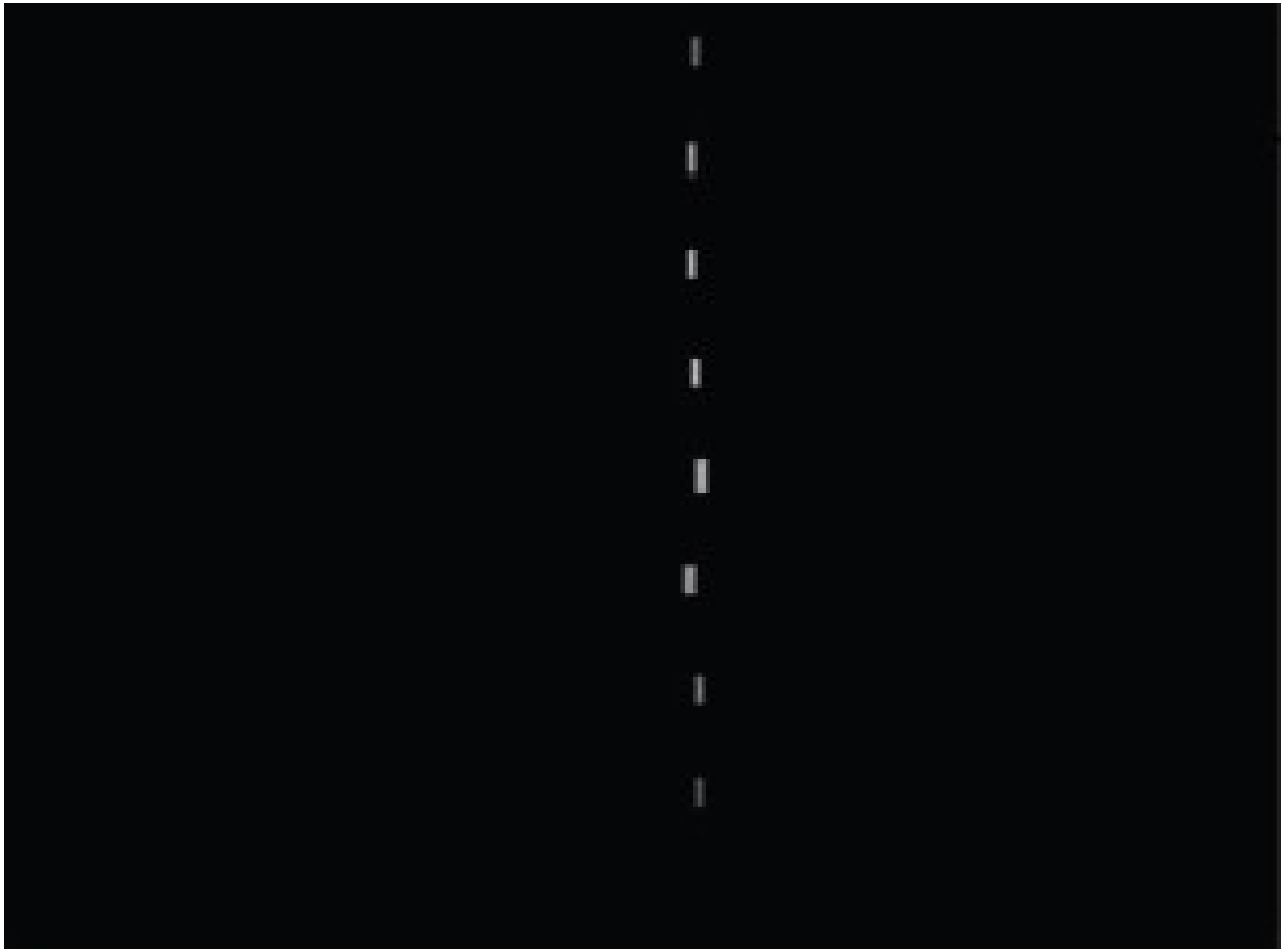
Spectral decomposition: first eigenvector



$$A_{i,j} x_i = \lambda x_i$$

A.Sarti,G.Citti 2011,2014

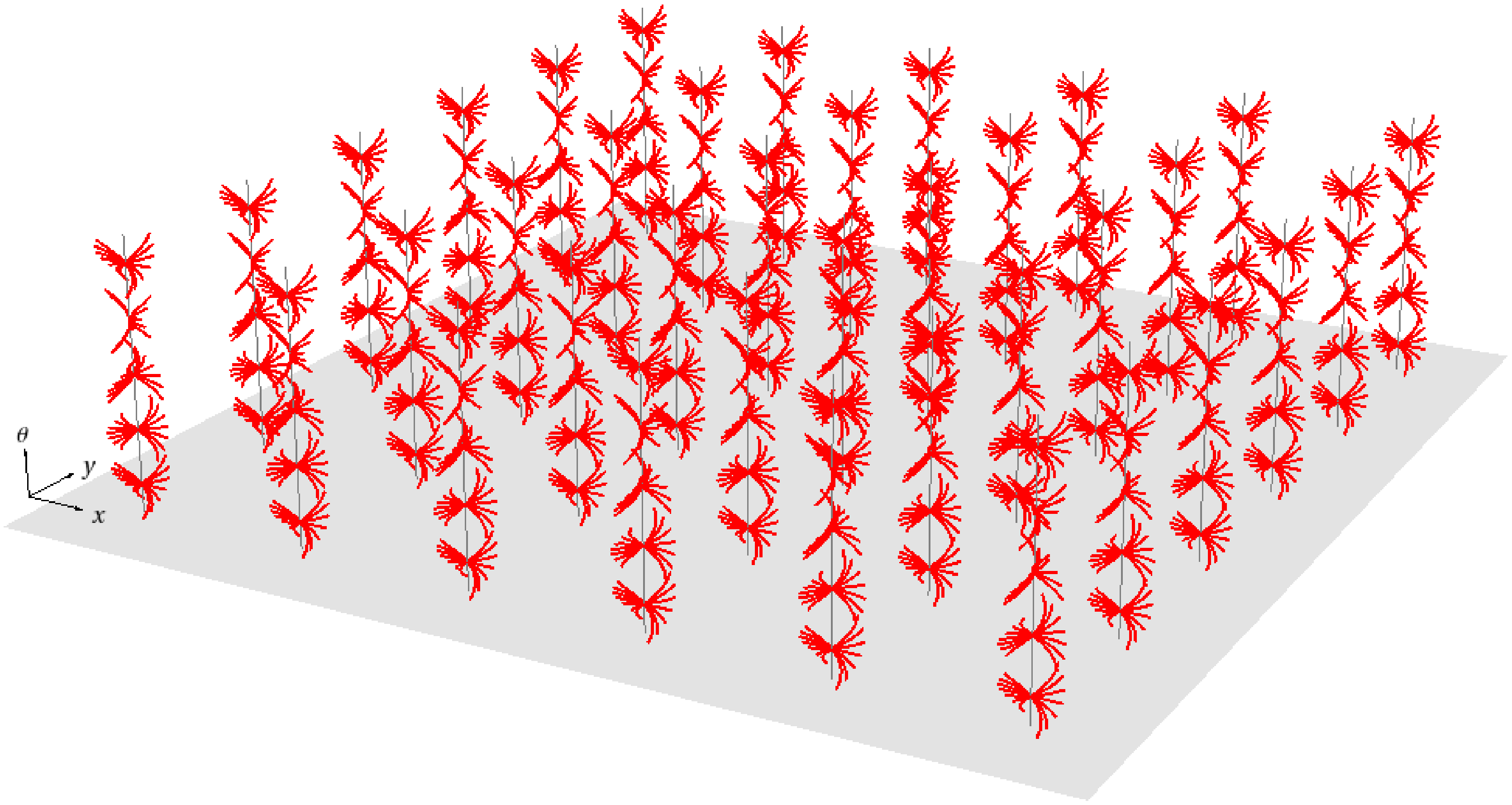
Spectral decomposition: 2nd eigenvector



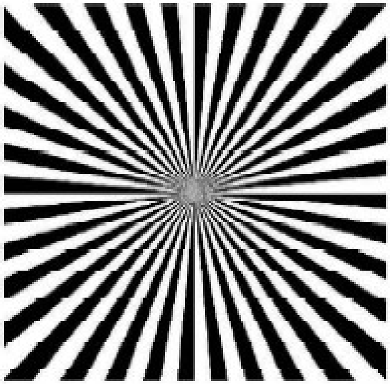
$$A_{i,j}x_i = \lambda x_i$$

A.Sarti,G.Citti 2011

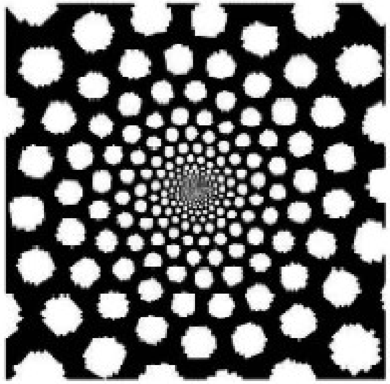
The full neurogeometrical graph



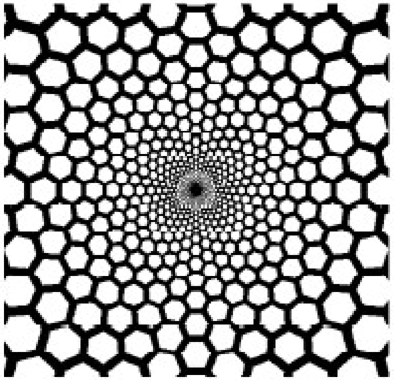
Eigenvectors of the full SE(2) structure: visual field



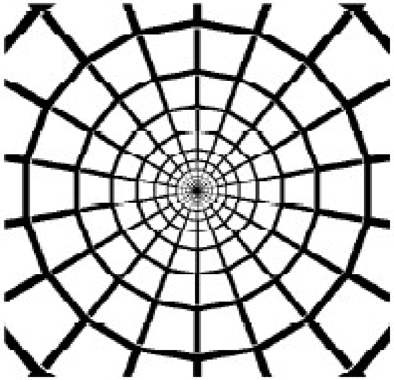
(a)



(b)



(c)



(d)

Heterogenesis

A zoo of groups

LGN Mexican hat   T(2)

Simple cells   SE(2)

Complex cells Engel

Non separable complex cells Galilean

Stereo SE(3)

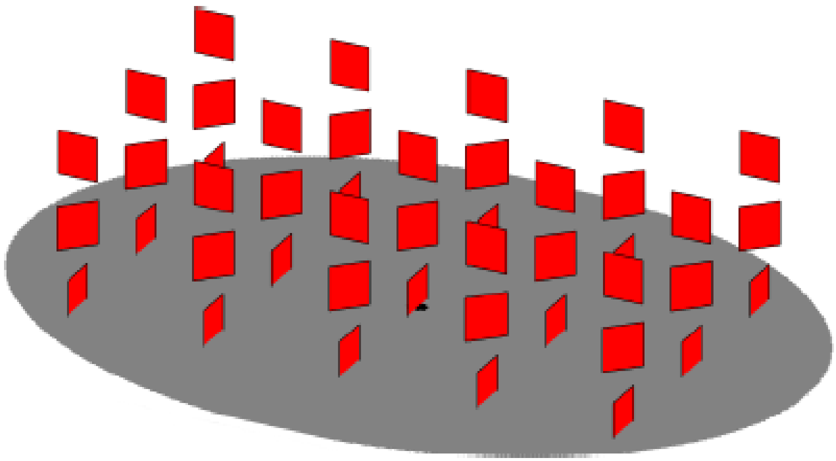
$$p \in B_{p_0}$$

$$A_{p_0}(u)(p) = A_{p_0}(p, u(p), \nabla_{p_0} u(p), \nabla^2_{p_0} u(p), \cdots, \nabla^k_{p_0} u(p))$$

$$\nabla_{p_0} = (X_{1,p_0} X_{2,p_0}, \cdots)$$

$$\tilde{\nabla}_{p_0} = (X_{1,p_0} X_{2,p_0}, \cdots, [X_{i,p_0}, X_{j,p_0}])$$

$$\tilde{A}_{p_0}(u)(p) = A_{p_0}(p, u(p), \tilde{\nabla}_{p_0} u(p), \tilde{\nabla}^2_{p_0} u(p), \cdots, \tilde{\nabla}^k_{p_0} u(p))$$



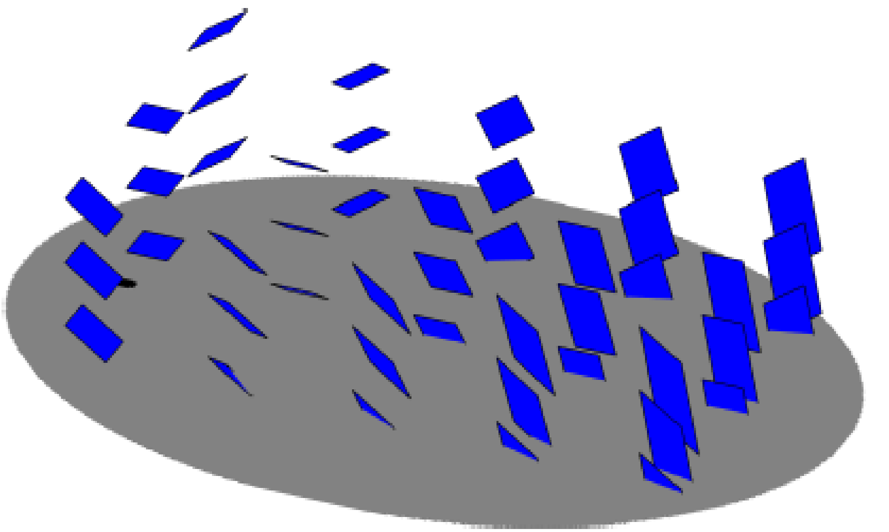
$$p \in B_{p_1}$$

$$A_{p_1}(u)(p) = A_{p_1}(p, u(p), \nabla_{p_1} u(p), \nabla^2_{p_1} u(p), \cdots, \nabla^k_{p_1} u(p))$$

$$\nabla_{p_1} = (X_{1,p_1} X_{2,p_1}, \cdots)$$

$$\tilde{\nabla}_{p_1} = (X_{1,p_1} X_{2,p_1}, \cdots, [X_{i,p_1}, X_{j,p_1}])$$

$$\tilde{A}_{p_1}(u)(p) = A_{p_1}(p, u(p), \tilde{\nabla}_{p_1} u(p), \tilde{\nabla}^2_{p_1} u(p), \cdots, \tilde{\nabla}^k_{p_1} u(p))$$



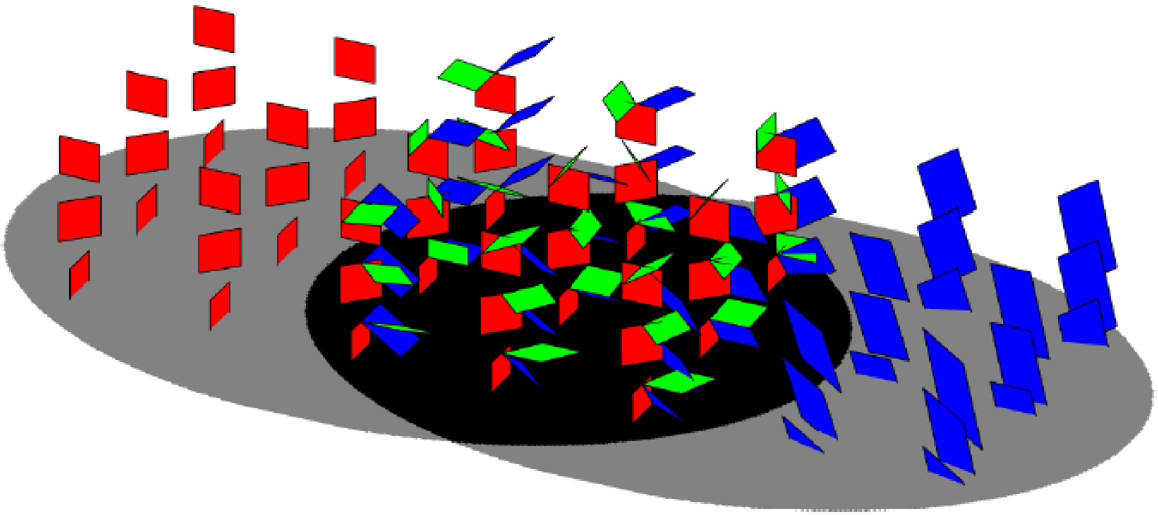
Heterogenesis

$$B_{p_0} \cap B_{p_1}$$

$$\nabla_{p_0,p_1} = (\nabla_{p_0}, \nabla_{p_1})$$

$$\tilde{\nabla}_{p_0,p_1} = (\tilde{\nabla}_{p_0}, \tilde{\nabla}_{p_1}, [\tilde{\nabla}_{p_0}, \tilde{\nabla}_{p_1}])$$

$$\& \tilde{A}_{p_0,p_1} := \phi_{p_0} \tilde{A}_{p_0} + \phi_{p_1} \tilde{A}_{p_1}$$



A.S, G.Citti, D.Piotrowski 2018 in press